Unravelling Strings at the LHC

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We construct LHC signature footprints for four semi-realistic string/M theory vacua with an MSSM visible sector. We find that they all give rise to limited regions in LHC signature space, and are qualitatively different from each other for understandable reasons. We also propose a technique in which correlations of LHC signatures can be effectively used to distinguish among these string theory vacua. We expect the technique to be useful for more general string vacua. We argue that further systematic analysis with this approach will allow LHC data to disfavor or exclude major "corners" of string/M theory and favor others. The technique can be used with limited integrated luminosity and improved.

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I. INTRODUCTION

The progress of string theory in the last decade has brought us closer to the ambitious goal of connecting string theory to reality and testing it in various experiments. However, developments in the past few years seem to suggest that instead of predicting a *unique* well-defined vacuum from some underlying dynamical principle, string theory gives rise to a vast "landscape" of string vacua. From a particle physics perspective, this implies the existence of a vast class of effective theories for beyond-the-Standard-Model physics based on different choices of string compactifications to four dimensions. Nevertheless, we would like to learn more about string vacua, particularly about aspects soon to be illuminated by LHC data.

If one is interested in connecting string theory with reality, it is important to know if it is possible to differentiate the effective theories arising in string compactifications from each other based on real experimental observables, such as LHC signatures. This question was investigated in [1], where it was argued that specific string constructions usually lead to a specific pattern of LHC signatures. In [1], the general idea and a simple method to differentiate different classes of string constructions was proposed. In this paper we continue our exploration along this direction. The goals of this paper are two-fold. The first is to demonstrate convincingly that the "footprint" of a well-defined class of string constructions is limited, so in particular it is not the case that any arbitrary signature is compatible with these "stringy" effective theories. The second is to propose a systematic technique based on the correlation of signatures to tell whether two classes of constructions can be distinguished or not.

Suppose the LHC detector groups report a signal beyond the Standard Model (SM). We expect and assume here that experimenters and SM theorists will get that right. We want to focus on interpreting the data in terms of an underlying theory. Most work in this direction has tried to build a bottom-up approach by deducing which new particles are produced, and constructing an effective Lagrangian at the Electroweak (EW) scale. Such work should of course be pursued. But we have increasingly learned how difficult it may be because of issues like large number of parameters [2], degeneracies [3], etc., so complementary alternative approaches are good.

If we knew the underlying theory at the unification scale, it would be possible to express the many low scale effective theory parameters in terms of perhaps a few microscopic parameters and many degeneracies would disappear. Of course we do not know the correct underlying theory. We argue in the following that it may be possible to overcome this by studying a number of classes of underlying theories and by systematically using the pattern and correlations of LHC signatures and related data. In a sense, we are arguing for a mapping of LHC data onto underlying theories.

Our approach can be used for any kind of underlying theory, at any scale. We prefer to work with string/M theory however, because we expect that it will be how nature is described. Within various string/M theory models, we want to work with those which have moduli stabilized so that reliable predictions can be made. Our attitude is that LHC signatures and related data depend on new particle masses and couplings and on the constraints imposed by the underlying theory, but in a very complicated way that is difficult to extract. By studying patterns of signatures [1, 4] we can learn the implications of the data. Insights from low scale effective theory analysis carried out in parallel can also be included in our analysis.

Ideally, we hope there is a progress when one not only compares different theories, e.g. Type II vs. heterotic etc., but also takes a given type of theory and compactifies several ways, for each compactification one can break supersymmetry several ways, etc. One can systematically study what kinds of data can distinguish them.

The basic idea of this paper is as follows. Based on the mapping from model parameter space to signature space, any Beyond-the-Standard-Model construction corresponds to a high-dimensional sub-manifold in the signature space, which we call the "footprint" of the construction. Sometimes we will be sloppy and also refer to a particular 2D slice as its footprint. This should be clear from the context. By taking into account the current experimental data, we may constrain the footprint. So, the shape, size and position of a

footprint carries non-trivial information about the original construction, which is encoded in the correlation of different signatures of the entire construction, and also about constraints from existing data. We develop a technique by which one could extract this information effectively and use it to distinguish different string-theoretical constructions. The constructions we study have already been studied in the literature, in particular calculations for them have been done. But for consistency we do our own calculations for all the models.

In this paper we do not focus on details of how one scans the microscopic parameters, their metric, SM and detector, background and fluctuations, etc. All of these kinds of issues should be treated in detail in application when there is data, and in a more computer intensive study that is underway, but they do not affect qualitative conclusions about footprints and distinguishing theories.

II. REALISTIC STRING VACUA

In order to be precise, we list the criteria required for a class of string vacua to be realistic. For concreteness, we only focus on string vacua with low-energy supersymmetry since it appears to be the most well motivated solution to the Hierarchy Problem; however realistic string vacua with other methods of explaining the Hierarchy can be similarly defined.

A. String-Susy Models

To qualify as what we call a "String-Susy Model", we require a class of string vacua arising from a compactification to four dimensions to have the following properties:

- It has $\mathcal{N}=1$ supersymmetry in four dimensions which is broken in a controlled approximation.
- The moduli are stabilized in a metastable dS vacuum and a stable hierarchy between the Electroweak and Planck Scales is generated.
- The visible sector accommodates the MSSM particle content and gauge group (maybe with additional matter and gauge groups) and their properties.
- It has a mechanism for breaking the Electroweak symmetry.
- It is consistent with all experimental constraints.

In addition, the fact that gauge couplings in the MSSM unify with great precision at $\sim 2 \times 10^{16}$ GeV seems to be tantalizing evidence for gauge coupling unification and a unified theory framework. Although some string-susy models give rise to gauge coupling unification naturally and some don't (for example, LARGE Volume models do not give rise to gauge coupling unification with an MSSM visible sector), gauge coupling unification is still an important criterion in our opinion and should serve as an important guide for constructing realistic string theory vacua.

Present models do not quite meet these criteria, but are close enough to justify working with them. More precisely, we study four dimensional vacua in string theory where the first two conditions are met in a reliable manner, i.e. the question of supersymmetry breaking, moduli stabilization and generation of the Hierarchy is answered in a convincing manner. One popular example is KKLT vacua proposed by Kachru, Kallosh, Linde and Trivedi [5]. The KKLT vacua do not comprise a string-susy model in the strict sense since an explicit compactification using the KKLT mechanism to stabilize the moduli and containing a visible sector satisfying the last two conditions has not yet been constructed. However, KKLT vacua still allow a "prediction" if a certain visible sector particle content is assumed. For example, if the particle content is assumed to be that of the MSSM, it picks out a subset of MSSM models which encodes features of KKLT vacua in their spectra and signature pattern. This procedure can be carried out for other classes of vacua as well. One thus obtains classes of models, each of which are completely specified by a set of properly chosen stringy/microscopic parameters characterizing the particular class of string vacua. By a slight abuse of notation, we will still call them "string-susy models". The consequences of such MSSM models at the LHC can be readily predicted by standard methods. For simplicity and concreteness, in this paper we assume that an MSSM visible sector particle content is realized for each string-susy model. It would be very interesting to relax this requirement in the future and study the consequences. When concrete matter embeddings are available in the above classes of models, there may arise extensions of the MSSM. It will be interesting to study them.

In this paper, we study three well-motivated classes of string vacua assuming an MSSM visible sector - Type IIB KKLT vacua [5, 6], Type IIB LARGE-Volume vacua [13] and fluxless M theory vacua [17, 18] on G_2 manifolds. For KKLT vacua, we also study a variation of the original KKLT procedure which uses the mechanism of F-term uplifting rather than that by anti D3-branes [20]-[25]. Each of these vacua have been studied in the literature in detail and the interested reader can consult the relevant references. Next we briefly summarize their most important features.

If an MSSM visible content is realized in these constructions, a set of soft supersymmetry breaking parameters will be generated after supersymmetry breaking. These MSSMs together with a set of soft supersymmetry breaking parameters then constitute our "stringsusy MSSMs".

B. Description of String-Susy Models

1. (Original) KKLT MSSM vacua - SUSY breaking by \overline{D} 3-branes (KKLT-1)

This class of constructions is a part of the IIB landscape with all moduli stabilized [5]. Closed string fluxes are used to stabilize the dilaton and complex structure moduli at a high scale and non-perturbative corrections to the superpotential are used to stabilize the lighter Kähler moduli. One obtains a supersymmetric anti-deSitter vacuum. The hidden sector is an anti-D3-brane at the IR end of the throat, and is thus sequestered from the visible sector. The anti D3-brane breaks supersymmetry as well as lifts the vacuum to a

deSitter one. Supersymmetry breaking is then mediated to the visible sector by gravity. The flux superpotential (W_0) has to be tuned very small to get a gravitino mass of $\mathcal{O}(1\text{-}10\text{ TeV})$. The soft supersymmetry breaking terms at the unification scale are calculated in [6]:

$$M_a = M_s \left[l_a \alpha + b_\alpha g_a^2 \right],$$

$$m_i^2 = M_s^2 \left[(1 - n_i) \alpha^2 + 4\alpha \xi_i - \dot{\gamma}_i \right],$$

$$A_{ijk} = -M_s \left[(3 - n_i - n_j - n_k) \alpha - \gamma_i - \gamma_j - \gamma_k \right],$$
(1)

where b_a are the β function coefficients, γ_i is the anomalous dimension and $\dot{\gamma}_i = 8\pi^2 \frac{\partial \gamma_i}{\partial \ln \mu}$. The coefficient ξ_i is a complicated function of trilinear couplings, Yukawa couplings and gauge couplings [6]. Here $M_s \equiv m_{3/2}/(16\pi^2)$ characterizes the size of the AMSB contribution and α is the ratio of the modulus-mediated contribution to the AMSB contribution, defined as in [10] ¹. The parameter α is determined by the form of the uplifting potential and the flux contribution. In [12], it was argued that typically α can take a generic value of order unity, which in the definition of [10] is of order $16\pi^2/\ln(M_p/m_{3/2}) \sim 5$ for $m_{3/2} = (1-10)$ TeV.

The SM gauge fields can live on D7-branes or D3-branes, which corresponds to $l_a = 1$ or 0 respectively. We will focus on the former case and set $l_a = 1$. The chiral matter fields can be constructed by adding intersecting D7-branes with magnetic fluxes in their worldvolume. In the case of toroidal (orbifold) compactifications and no magnetic fluxes, the modular weights n_i can take values 0, 1/2 or 1 depending on whether the matter fields are on the D7brane, D3-D7 intersection or D3-brane respectively. For compactifications with more general Calabi-Yau manifolds or with more general intersecting D7-brane models with worldvolume magnetic fluxes, n_i will be model dependent and have to be computed in each model[11]. Generally if the modular weighs are equal to 1, all the scalars will be tachyonic as a result of the mixing of the moduli and AMSB contribution in Eq.(1). So modular weights of 1 are normally excluded. In addition, the ratios of gaugino masses at low scale are roughly $(1+3.3/\alpha):(2+1/\alpha):(6-9/\alpha)$. For a typical value of $\alpha=5$, the ratio is 1.5: 2: 3.8. The LSP is predominantly bino-like for a sizable range of α around 5. The constraint on the relic density of neutralino dark matter favors the region in the parameter space where there is some bino-wino mixing (not necessarily large), or, a stop or stau with mass close to that of LSP.

2. KKLT MSSM vacua - SUSY breaking by hidden sector F-terms (KKLT-2)

There is a variation of the original KKLT proposal in which the anti D3-brane is replaced by a hidden sector which spontaneously breaks supersymmetry and lifts the AdS minimum. This is also known as F-term uplifting. Several examples of this type of vacua are discussed in the literature [20]-[25]. A notable example is to use the recently discovered ISS model [32] as the hidden section, which can potentially have a dual stringy construction via AdS/CFT

¹ Note this definition is different from that in [12].

duality [26]. In this example, dS vacua with zero cosmological constant and TeV scale gravitino mass can both be realized naturally at the same time [24].

The phenomenology associated with this class of vacua is model-dependent and is still under investigation. To our knowledge, a generic parametrization of soft supersymmetry breaking terms can be found in [25] and will be used in our analysis. In this result, the gaugino masses and trilinears are similar to those in the original KKLT proposal, while the scalar masses are of the form:

$$m_i^2 = (16\pi^2 M_s)^2 (1 - 3\zeta_i). (2)$$

Here ζ_i are the couplings entering the matter Kähler potential:

$$K_{\text{matter}} \sim \bar{Q}_i Q_i (T + \bar{T})^{n_i} \left[1 + \zeta_i \bar{\phi} \phi + \mathcal{O}(\phi^4) \right].$$
 (3)

where Q_i are the visible sector matter fields and ϕ is the hidden sector matter field. In general without special assumption on the construction (e.g. geometric separation), the hidden sector is not sequestered from the visible sector. So ζ_i are expected to be of order unity, which gives rise to unsuppressed scalar masses ($\sim m_{3/2}$). Thus the modular weights n_i are not important in determining soft terms and are set to zero for convenience. In the limit $\zeta_i \to 1/3$, the scalars become light and the mirage pattern of scalar masses is recovered. In addition, the non-sequestering of the hidden sector also implies a larger range of values of α [25]. In our phenomenological analysis, this class of models will be referred to as KKLT-2.

3. LARGE Volume MSSM vacua (LGVol)

This class of constructions also form part of the IIB landscape with all moduli stabilized. In this case, the internal manifold admits a large volume limit with the overall volume modulus very large² and all the remaining moduli small [13]. Fluxes again stabilize the complex structure and dilaton moduli at a high scale, but the flux superpotential W_0 in this case can be $\mathcal{O}(1)$. To stabilize Kähler moduli in the large volume region, one needs to incorporate the perturbative contributions (α' correction) to the Kähler potential as well as the non-perturbative contributions to the superpotential since they are equally important. The AdS minimum of the resulting potential is already non-supersymmetric in contrast to the KKLT case, which can be lifted to a de Sitter one by similar mechanisms as in the KKLT case.

This class of vacua turns out to be more general and includes the KKLT vacua as a special limit, in which W_0 is tuned very small [13, 15]. However, when W_0 is $\mathcal{O}(1)$, the conclusions are qualitatively different. We will analyze such a situation, since then there

² We distinguish between what is usually called large volume, where manifold volumes are several times the volume in Planck units, and the volumes of manifolds for these models where the volume is several orders of magnitude larger than the volume in Planck units, giving rise to an intermediate scale string scale. We denote the latter case by LARGE volume.

will be no theoretical overlap between the two classes of vacua. The exponentially large volume V generated allows both lowered string scale and gravitino mass

$$m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}, \quad m_{3/2} \sim \frac{M_P}{\mathcal{V}}.$$
 (4)

To get a TeV-scale gravitino mass, one needs a volume $V \sim 10^{15}$ which gives rise to an intermediate string scale $m_s \sim 10^{11}$ GeV. Since the string scale is much smaller than the unification scale, one cannot have the standard gauge unification in these compactifications with only MSSM matter.

In this class of constructions, the Standard Model sector arises from an appropriate configuration of D7-branes which wrap a small four-cycle (a four dimensional submanifold of the entire Calabi-Yau manifold) corresponding to the modulus τ_s . To generate chirality, the SM D7-brane is required to be magnetized, which gives rise to a modified gauge kinetic function

$$f_i = \frac{T_i}{4\pi} + h_i(F)S,\tag{5}$$

where h_i is a topological function of the magnetic flux on the brane. An exact calculation of the gaugino masses at the lowered string scale gives the following boundary condition

$$M_1: M_2: M_3 = k_Y g_1^2: g_2^2: g_3^2,$$
 (6)

where k_Y is determined by the normalization of the U(1) charge. Since there is no gauge coupling unification in this construction, the gaugino masses are also not unified at the string scale.

The scale of the gaugino masses is determined by the F-term F^s of the small four-cycle, which is characterized by

$$M_c \equiv \frac{F^s}{2\tau_s} \approx \frac{1}{2}(M_2 + M_3),\tag{7}$$

where $\tau_s = \text{Re}(T_s)$ is the modulus associated with the small four-cycle. There is a so-called "dilute flux limit" in which the magnetic flux is diluted by increasing the size of the large four-cycle. In such a case, it was shown in [14] that the scalar and trilinear terms at high scale take simple expressions and are given by:

$$m_i = \frac{1}{\sqrt{3}} \frac{F^s}{2\tau_s} = \frac{M_c}{\sqrt{3}} \tag{8}$$

$$A_{ijk} = -\frac{F^s}{2\tau_s} = -M_c. (9)$$

Generally the presence of the fluxes will modify the above equations. The effects of these fluxes are modelled by small perturbations ϵ_i around the above results. Therefore the LARGE Volume soft spectrum can be parameterized as in [14]

$$M_{1} = M_{c}(1 + \epsilon_{1})$$

$$M_{2} = M_{3}/1.37$$

$$m_{i} = M_{c}(1 + \epsilon_{i})$$

$$A_{ijk} = -\frac{1}{\sqrt{3}}(m_{i} + m_{j} + m_{k}),$$
(10)

where ϵ_i were randomly generated within a domain $0 < \epsilon_i < \epsilon_0$. A reasonable value for ϵ_0 can be taken to be 0.2 as in [14]. The low scale gaugino mass ratios calculated from the above boundary conditions are (1.5-2):2:6. As we see, the ratio of M_2 and M_3 is fixed but the ratio of M_1 and M_2 or M_3 is not completely fixed. This can be seen from (10) as open string fluxes lead to uncertainties for M_1 . At high scale, gaugino masses and squark masses are roughly the same, and are both boosted by the SU(3) interaction when RG evolved to the low scale. So generically the gluino is the heaviest particle and the first and second generation squarks are only a little lighter than the gluino. The tau slepton is extremely light (close to the mass of the LSP) which is needed to not overclose the universe by the bino LSP relics.

4. Fluxless M theory
$$G_2$$
-MSSM vacua (G_2)

The M-theory vacua we consider here follow reference [17]-[19]. One studies fluxless M theory compactifications on G_2 manifolds with at least two hidden sectors undergoing strong gauge dynamics, at least one of which has charged matter. This leads to a stabilization of all moduli and the spontaneous breaking of supersymmetry. The supersymmetry breaking is dominated by the hidden sector meson field ϕ , which is not sequestered from the visible sector. The gauge kinetic function is a linear combination of all geometric moduli s_i . For the case where the matter Kähler metric does not depend on ϕ , the gaugino masses receive comparable contributions from moduli and anomaly mediation, but is different from mirage mediation in the KKLT string-susy model. The high scale gaugino masses have the following form:

$$M_a \approx -\frac{1}{4\pi(\alpha_M^{-1} + \delta)} \left\{ b_a + \left(\frac{4\pi \alpha_M^{-1}}{P_{\text{eff}}} - b_a' \phi_0^2 \right) \left(1 + \frac{2}{\phi_0^2 (Q - P)} \right) \right\} m_{3/2} \quad (11)$$
where $b_1 = 33/5$, $b_2 = 1.0$, $b_3 = -3.0$, $b_1' = -\frac{33}{5}$, $b_2' = -5.0$, $b_3' = -3$.

 α_M is the tree-level universal gauge coupling and δ is the threshold correction from the Kaluza-Klein modes. ϕ_0 is the vev of the meson field, which is of order unity. P and Q are the ranks of the hidden sector gaugino condensation groups. Low scale supersymmetry can be obtained only if Q - P = 3. To get dS vacua, it is also necessary that the combination $P_{\text{eff}} \equiv P \ln(A_1 Q/A_2 P)$ is less than 84, and larger than about 50 to get a gravitino mass below 100 TeV. The scalar masses are about equal to $m_{3/2}$ as there is no sequestering in general. Thus, the soft supersymmetry breaking pattern is such that there is a large mass splitting between gauginos and scalars, and the low energy effective theory at the weak scale is mainly determined by the gaugino sector. Unlike split-SUSY, the higgsinos in these vacua are as heavy as scalars and also decoupled. This gives the low scale gaugino masses large finite threshold corrections from the higgs-higgsino loop. Generically the wino is the LSP for G_2 -MSSM models with light spectra, but a wino-bino mixture is also allowed particularly for heavier spectra.

The three classes of type IIB MSSM vacua we described above are related in some ways. As seen in Sections IIB1 and IIB2, KKLT-1 and KKLT-2 are different in the supersymmetry breaking and uplifting mechanism and they generically give rise to different soft supersymmetry breaking terms. KKLT-2 in some sense a broader class of constructions as the explicit structure of the hidden sector is not completely specified, while that in KKLT-1 is completely specified. Thus it is in principle possible to make a model within the KKLT-2 class which has exactly the same soft supersymmetry breaking terms as KKLT-1. This happens for example when the hidden section is sequestered from the visible section and ζ_i goes to 1/3 as has been already shown. So, in terms of the soft susy breaking pattern, models of KKLT-1 are a subset of those of KKLT-2. One should keep in mind that this overlap is a "theoretical overlap" and cannot be distinguished at the LHC. So in the distinguishibility analysis in the rest of the paper, we will focus on the region of KKLT-2 which does not overlap with KKLT-1. Further study is needed to learn whether cosmological or additional visible sector physics can distinguish these constructions phenomenologically.

The KKLT and LARGE Volume MSSM vacua (described in IIB1 and IIB3) are two distinct regions in the type IIB landscape. However they can be smoothly connected by dialing some parameters. The same scalar potential which gives rise to the large-volume minimum also has a KKLT minimum if $W_0 \ll 1$ [13]. As W_0 decreases, the two minimums approach each other and eventually merge. So one could in principle start with LARGE Volume MSSM vacua and decrease W_0 while keeping $m_{3/2}$ fixed by decreasing the volume \mathcal{V} . In this way, the large-volume minimum will gradually lose its "large volume" property and become more and more like a KKLT vacua with TeV scale gravitino mass. We do not know much about the properties of these intermediate vacua. One possibility is that they lead to a set of soft supersymmetry breaking terms which interpolate in between the LARGE Volume MSSM vacua and KKLT MSSM vacua. Even if this is true, phenomenologically it is not clear whether these intermediate vacua will survive after all kinds of experimental and consistency constraints. It may be that a continuous set of intermediate vacua consistent with data and theory do not exist.

III. FOOTPRINT OF "STRING-SUSY MODELS" AT THE LHC

A. How to construct a Footprint in general

As was explained in section IIA, a string-susy model is specified by a set of microscopic parameters characteristic of the class of string vacua. A complete analysis for the whole (microscopic) parameter space is necessary if one hopes to discriminate between different string-susy models. The prediction of a given string-susy model at the LHC is a map from this parameter space to LHC signature space. This is a multi-dimensional region which we call the "footprint" of the particular string-susy model. In this paper we construct

 $^{^{3}}$ in the sense that the two class of models give rise to the same soft terms from a theoretical point of view.

a footprint for the three string-susy models described earlier - KKLT MSSM vacua (two variations), LARGE Volume MSSM vacua and G_2 -MSSM vacua. We first start with a general discussion of constructing a footprint of any string-susy model.

As seen in the last section, a set of MSSM soft supersymmetry breaking parameters can be obtained at the compactification scale⁴. Below this scale, heavy stringy and Kaluza-Klein states decouple and the soft parameters of the MSSM fields are governed by the MSSM renormalization group equations. Gauge couplings and Yukawa couplings are determined from current experimental data. The μ parameter is determined by the correct electroweak symmetry breaking and Z boson mass. It would of course be preferable to calculate μ and $\tan \beta$ from the microscopic theory as well, but that is not yet possible. In the G_2 case $\tan \beta$ is calculated from the theory, but not μ .

In addition the low scale soft supersymmetry spectrum is subject to various constraints from current observation. There are lower bounds on the masses of the various sparticles from the SUSY searches at LEP and Tevatron. The most important ones are the chargino mass limit and the higgs mass limit. There are also constraints from observations in cosmology, i.e. dark matter relic abundance Ωh^2 . Although one can compute the thermal relic density reliably, there may be other contributions from non-thermal production or other unknown sources, so one should impose an upper limit constraint but not a lower limit one.

In order to connect to LHC experiments, the next step is to simulate the pp collision and the decay of particles produced at the LHC followed by detection of the surviving particles in the final state. In our analysis, we use the PGS4 [31] package which generates events using PYTHIA6.4 [28] and then perform the detector simulation, where the default configuration of the detector parameters are used. While PGS is not a fully realistic detector simulator for the LHC, it is simple, fast and gives a "pretty good" simulation result, as its name suggests. The result from PGS usually agrees fairly well with the result one might obtain with a full-fledged detector simulation. In many cases the agreement is good, of the order of 20%. To construct the footprint of the string-susy models described earlier, we sample the high-scale (microscopic) parameter space with a large number of points. For collider phenomenology, the simplest assumption of equal probability distribution on the parameter space is presumably sufficient. For each point we sample, the corresponding signatures are computed through the aforementioned procedure. For our purposes here, where we compare predictions of different models, these procedures are adequate. Later the analysis can be sharpened. The procedure to go from string/M theory to signatures may seem complicated, but now user-friendly softwares exist to do that. One can access much of the software through the LHC Olympics website [27].

To demonstrate the general approach shown above, we construct footprints of the four string-susy models discussed for an integrated luminosity of 5 fb^{-1} . In the simulation, we use the L2 trigger in PGS to get better S/B ratios [27]. For signatures, we use the following selection cuts for objects in each event

• Jet $P_T > 50$ GeV; Lepton and Photon $P_T > 10$ GeV; $\not\!\!E_T > 100$ GeV.

⁴ This is typically the unification scale

This means only objects satisfying the above cuts are kept in the event record. For backgrounds, we use the background sample in the LHC Olympics webpage [27] which includes dijets, $t\bar{t}$ and W, Z+jets processes and scale it up to get an estimate of the background for an integrated luminosity of 5 fb^{-1} . Other backgrounds may be important and should be taken into account in a more thorough analysis of the backgrounds, but as will be seen below the treatment of backgrounds will not have much affect on our main results. The condition for a (counting) signature to be observable above the background is:

$$\frac{S}{\sqrt{B}} > 4, \quad S > 5,\tag{12}$$

where S is the number of signal events that pass the selection cuts, while B is number of background events that pass the same cuts. Thus we can assign an observable limit for each (counting) signature below which the signal is not likely to be observed.

Figures 1 and 2 represent some simple 2D slices of footprints. There is no particular reason for the above choice of signature plots, they are just meant to illustrate general features. Some of these plots however have the added advantage that they are also useful in distinguishing some string-susy models. In all the (counting) signature plots, the approximate regions where the SM dominates are entirely blacked out ⁵. Immediately one can see from these plots that the footprints for these string-susy models are finite regions in signature space. This implies that a well-defined string-susy model is not likely to cover the whole signature space, but only a part of it. In addition, based on these footprints, one can readily distinguish among the string-susy models in many cases. For example, the plot of 1-b jet vs. 3-b jets clearly separates the KKLT-1 and G_2 string-susy MSSMs. By definition a footprint covers all possible signatures that might come out from a string-susy model. Therefore plots of footprints demonstrate the overall difference between different string-susy models. In this sense the footprint analysis generalizes the familiar benchmark analysis, where one does not get an overall picture of signatures for a given model thereby making it difficult to distinguish two classes of models. We emphasize that a (n-dim) footprint is the full region on a (n-dim) signature plot or any 2-D slice generated as the microscopic parameters are varied over their entire allowed ranges respectively.

B. Generic Features of Footprints

Some generic features of these footprints can be easily understood as follows. For simplicity, we will only focus on simple counting signatures, which illustrate many of the important points we want to emphasize. Counting signatures are always bounded by the maximum cross section, which is related to existing lower limits on masses. Hence the 2D projection of a footprint for counting signatures must be bounded along the radial direction. In addition, if no upper bound is imposed on sparticle masses, the footprint can continuously approach the origin. However the region below the observable limit is not interesting.

⁵ More precisely, it is the region bounded by the observable limits

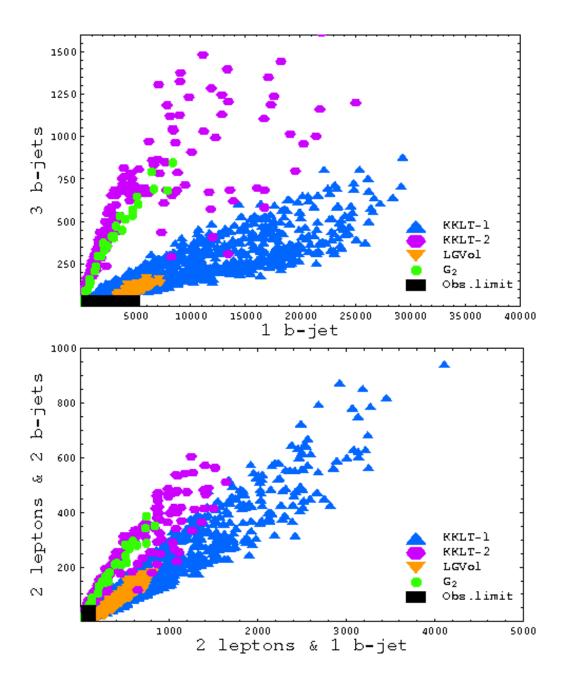


FIG. 1: Two-dimensional slices of the footprint of the three string-susy models. All models are simulated with $5fb^{-1}$ luminosity in PGS4 with L2 trigger. If not explicitly stated, all signatures include a least two hard jets and large missing transverse energy. For each example, the points are generated by varying the microscopic parameters over their full ranges, as explained in Section III D.

The angular dispersion of the footprint is due to the variation in the spectrum, which leads to a variation in branching ratios and in turn, the signatures. The smaller the angular dispersion the larger the correlation in the low scale soft spectrum and the more predictive the string-susy model. However, the exact spread depends on the particular signatures used because of many factors. For example, even a completely random MSSM soft spectrum will not cover the entire angular range from 0 to $\pi/2$. One also has to take into account real-

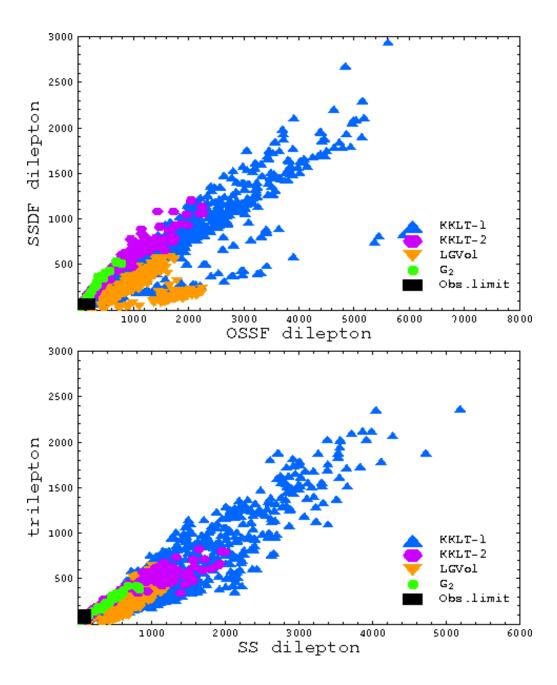


FIG. 2: Two-dimensional slices of the footprint of the three string-susy models. All models are simulated with $5fb^{-1}$ luminosity in PGS4 with L2 trigger. If not explicitly stated, all signatures include a least two hard jets and large missing transverse energy. For each example, the points are generated by varying the microscopic parameters over their full ranges, as explained in Section III D.

world "detector effects". For example, even for a model with no b-quark produced at parton level, there could be some b-jets in the final data since other quarks could be mistagged as b-quarks. For PGS4 loose b-tag, the charm quarks are mistagged as b-quarks with a probability 13%, while for other quarks the probability is about 1%. This implies that the ratio of b-jets to jets is at least 1%. In cases with significant charm quark production, the fake b-jets will be even larger. Another example is that a k-quark final state at parton level

can be read-off as an event with k-1 jets if one of the jets is soft and thereby fails to pass selection cuts, or if two jets merge together to form a single jet. The limited statistics of our simulations are not a problem since the precise regions are not needed for our main conclusions, and since we understand why the regions have the boundaries they do. Later, analysis with more statistics can be done.

C. Origin of Distinguishibility - Correlations

It is desirable and important to qualitatively understand the footprint boundaries and the difference between footprints. The features in footprints can be connected to the underlying theory by understanding correlations between soft parameters which in turn have their origin in the structure of the underlying theory. One could understand this as follows. Formally, the collider signatures (s_i) are functions of the MSSM masses and couplings (call them m_i in general), which are themselves parameterized by the underlying "microscopic" parameters (call them ξ_k). One has:

$$s_i = s_i(m_j) = s_i \Big(m_j(\xi_k) \Big). \tag{13}$$

For an arbitrary set of MSSM parameters, one would get a very broad set of signatures⁶, or equivalently, the corresponding footprint would cover a very large region in signatures space. However if there is a non-trivial dependence on the more fundamental (microscopic) parameters ξ_k , the MSSM parameters are correlated with each other and so are the signatures. Therefore by understanding how correlations between soft parameters are connected to the structure of the underlying theory, one can understand why a given footprint occupies a given region in signature space and not some other region. In order to make the task easier, it is helpful to first understand the footprint in terms of the pattern of spectra of the class of models and then understand the spectra in terms of the soft parameters determined from the underlying theory. For simplicity, here we only explain the former. The latter can also be done in a straightforward manner, the interested reader can refer to the references available for the string-susy models studied here. The features described here are not used in constructing the plots, but are valuable in understanding the plots.

Let us start with colored particle production. For KKLT-1 and LARGE Volume stringsusy models, the squarks are lighter than the gluino and the dominant production is squark pair production and the squark-gluino production. For G_2 -MSSM models, the dominant production is gluino pair production since all squarks are extremely heavy. The KKLT-2 models also have a large scalar mass and are dominated by gluino pair production. This difference in the dominant production channel already leads to a difference in the leptoncharge asymmetry, as seen in Figure 3. Below we list some broad distinguishing features in the spectra of the string-susy models and their related signatures.

1. Sleptons in KKLT-1 and LARGE Volume MSSM models are relatively light (lighter than the gluino). Moreover $\tilde{\tau}$ is generically the lightest slepton. On the other hand,

⁶ The signatures will still not be uncorrelated due to the structure of the MSSM itself and also due to detector effects. However, we are interested in correlations which are present in addition to these.

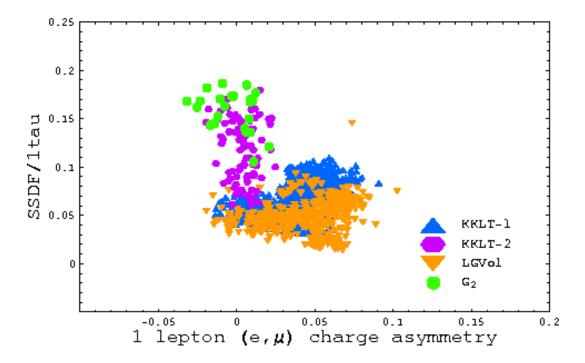


FIG. 3: A particular slice of footprint for the models studied. The one-lepton charge asymmetry (only include e and μ) is defined as $A_c^{(1)} \equiv \frac{N_l^+ - N_l^-}{N_l^+ + N_l^-}$. The SSDF/1tau signature^a is defined as the ratio of the number of events with SSDF dilepton and the number of events with 1 tau lepton. All models are simulated with $5fb^{-1}$ luminosity in PGS4 with L2 trigger. If not explicitly stated, all signatures include a least two hard jets and large missing transverse energy. For each example, the points are generated by varying the microscopic parameters over their full ranges, as explained in Section III D.

^aRatios of counting signatures are independent of the total rate and are sometimes useful. This gives an example.

sleptons in G_2 -MSSM models are very heavy (around $\mathcal{O}(10)$ TeV). So, signature plots sensitive to lepton flavor asymmetry could differentiate KKLT-1 and LARGE Volume from G_2 .

2. The gaugino mass ratios are different for different models, which lead to a difference in the jet multiplicity. For KKLT-1, the difference between M_3 and the LSP mass is much smaller than that of LARGE Volume and G_2 models (for the same gluino mass). So if we use a hard p_T cut on the jets (e.g. $P_T(jet) \geq 200$ GeV), then most of the four-jet events in KKLT-1 cannot pass the cuts since they are mostly from gluino pair production. However, for two-jet events, since the mass difference $m_{\tilde{q}} - M_{LSP}$ is large enough, most of them will pass the cuts. Thus we can probably use signature plots of events with 2 jets and events with 4 jets to distinguish models of KKLT-1 with those of LARGE Volume and G_2 . However in our plots, it can be seen there is still an overlap between KKLT-1 and LARGE Volume regions. The KKLT-1 models in the overlap region are exactly those with heavier gluinos.

- 3. Since the top Yukawa coupling is large, from RGE running the stop is lighter compared to other squarks. This is more pronounced for G_2 models since the $\tan \beta$ is particularly small (~ 1.5). The gluino will preferentially decay via a virtual stop and lead to b-rich events. For KKLT-1 and LARGE Volume models, the stop is again light and its production rate is big. However since all other squarks are also copiously produced, the overall branching ratio for the events with b-jets is not particularly large, so signature plots involving numbers of b-jets can distinguish G_2 models from those of KKLT-1 and LARGE Volume. In cases with very light stops, the stop production rate is very large, but they will decay into charm quarks instead of bottom quarks if the stop is lighter than \tilde{C}_1 7. Therefore these models will again give relatively small number of events with b-jets.
- 4. The KKLT-2 models appear in the signature space similar to G_2 models because they both have very heavy scalar masses. However they extend to a much bigger region in many plots and have some overlap with KKLT-1 models. Because of the large scalar masses, KKLT-2 models can be in the focus point region which is consistent with the dark matter constraints. This means in these models the LSP has a significant higgsino component. We also know that because of the higgsino mixture, gluinos tend to decay into b-jets through the large Yukawa couplings. As the consequence of the variation in the higgsino fraction, there is a large spreading in the b-jet signatures.

In Fig. 4 and 5, we show how the b-jet multiplicity affects the relative positions of these footprints. It can be seen that as the b-jet multiplicity increases, the footprints of G_2 and KKLT-2 models become isolated from the other two with a larger angular separation. This demonstrates that the b-jet multiplicity is related to a certain structure of the underlying theory. Thus, this is an example of a signature which is directly correlated with underlying structure of the theory and is particularly useful. For KKLT and LARGE Volume models, their relative position does not change much as the b-jet multiplicity changes, implying they have similar structure with regard to b-jet multiplicity. To summarize, the boundaries and the distinguishibility of footprints can be understood in terms of the spectra, and in turn in terms of the correlations between the soft supersymmetry breaking parameters determined by the underlying string-susy model.

Finally one should remember that although in principle we can have a large number of signatures, they are not orthogonal to each other. It was shown explicitly in [3] that for a set of MSSM models with 15 parameters, the effective dimensionality of signature space (with 1808 signatures) is only ~ 5 or 6. This means differences in the spectra may be lost in the process of mapping to signatures. This can also be seen in the various signature plots we have made, where only a few of them are quite different. The origin of this is related to the nature of hadron collision where signatures are usually polluted by large combinatorial backgrounds. Therefore figuring out analytically how to pick out mutually independent signatures which can distinguish classes of models is very difficult in general.

⁷ In this case, the decays $\tilde{t} \to b\tilde{C}_1$ and $\tilde{t} \to t\tilde{N}_1$ are kinematically closed.

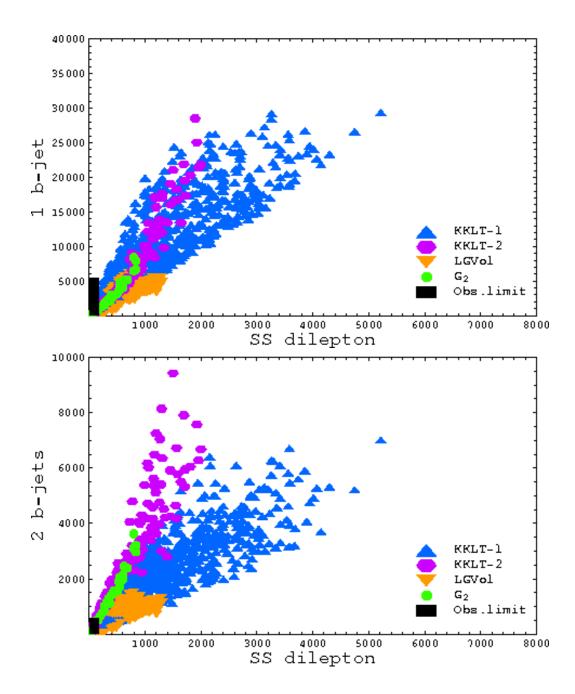


FIG. 4: Slices of footprints for the models studied. All models are simulated with $5fb^{-1}$ luminosity in PGS4 with L2 trigger. If not explicitly stated, all signatures include a least two hard jets and large missing transverse energy. For each example, the points are generated by varying the microscopic parameters over their full ranges, as explained in Section III D.

In practice, as demonstrated in Section IV, distinguishing classes of models can be done by adding lots of signature plots since the overlap region always decreases. By doing this one can identify useful independent signatures. Sometimes it also helps to pick signature plots sensibly based on the qualitative features described above. We will discuss more about this issue in Section IV A.

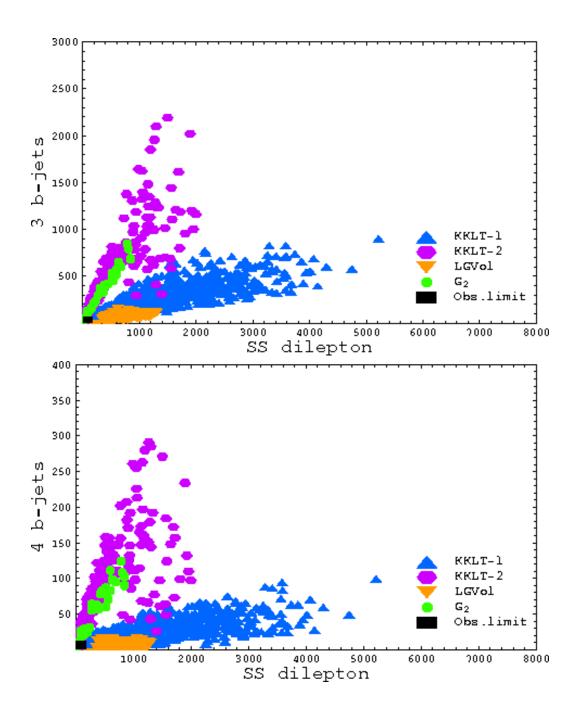


FIG. 5: Slices of footprints for the models studied. All models are simulated with $5fb^{-1}$ luminosity in PGS4 with L2 trigger. If not explicitly stated, all signatures include a least two hard jets and large missing transverse energy. For each example, the points are generated by varying the microscopic parameters over their full ranges, as explained in Section III D.

We have mostly focused on counting signatures, but one can also study various distributions. Distributions can be used similarly to counting signatures if they are converted into quantiles. In our initial analysis, we have implemented the following basic distributions:

• Effective mass of all objects $m_{eff} = \sum_a P_T^a$, divided into 12 categories labelled by

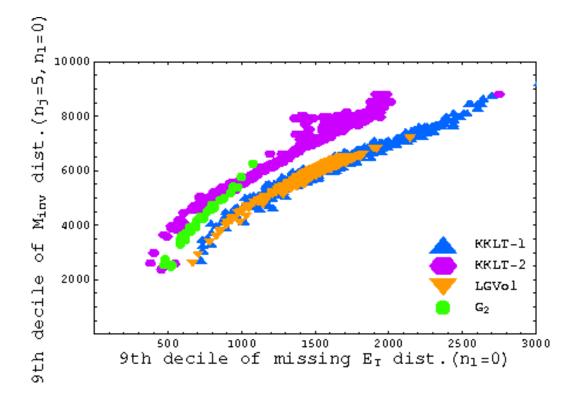


FIG. 6: Slices of footprints for the models studied. All models are simulated with $5fb^{-1}$ luminosity in PGS4 with L2 trigger. For each example, the points are generated by varying the microscopic parameters over their full ranges, as explained in Section III D.

number of jets and leptons in the event: $n_i = 2, 3, 4, 5^+, n_l = 0, 1, 2^+$

- Missing E_T distributions, divided into 3 categories labelled by the number of leptons $n_l = 0, 1, 2^+$
- Invariant mass of all objects $m_{inv} = \left(\sum_a P_\mu^a\right)^2$, divided into 12 categories labelled by number of jets and leptons in the event: $n_j = 2, 3, 4, 5^+, n_l = 0, 1, 2^+$

To get quantile (decile for example) signatures, the entries in a distribution are sorted into ten bins such that each bin contains 10% of the total events. The boundaries of the bins are taken as signatures, with no signatures related to the lower and upper boundaries of the whole distribution. Therefore each distribution gives 9 signatures and we have 243 signatures for the 27 distributions above. We have examined many different quantile signature plots. There are a few plots which can separate some string-susy models, two of them are shown in Figures 6 and 7.

We have seen not only that footprints are generally limited, but that we can always get a qualitative understanding of the boundaries of the footprint region. Therefore one can obtain a lot of insight even without a high statistics simulation of the footprint region in many cases, although in some cases when there is data increasing statistics could turn out to be important.

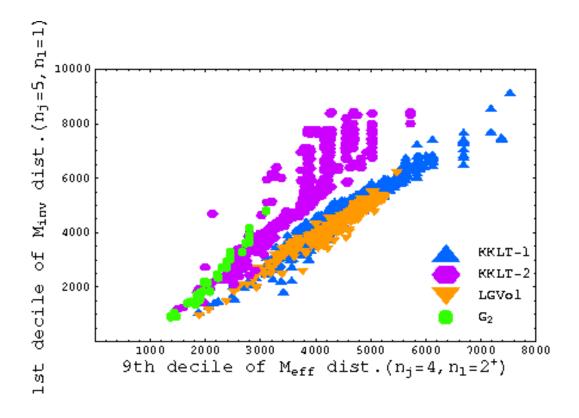


FIG. 7: Slices of footprints for the models studied. All models are simulated with $5fb^{-1}$ luminosity in PGS4 with L2 trigger. For each example, the points are generated by varying the microscopic parameters over their full ranges, as explained in Section III D.

D. Details of Constructing Footprints for String-Susy Models

In this subsection, we explain in more detail the construction of the footprint of the various string-susy MSSMs. Starting with the KKLT-1 string-susy model, the high scale parameter space is defined in section 2. Explicitly, the low scale soft spectrum is specified by the following parameters:

$$M_s, \quad \alpha, \quad n_l, \quad n_q, \quad n_h, \quad \tan \beta, \quad \operatorname{sgn}(\mu).$$
 (14)

To sample the parameter space, one has to set the ranges for scanning these parameters. First, based on the previous works [7, 8, 9, 10] we already know that for α below 4-5, the model is excluded either by the presence of tachyons or a stop/stau LSP, while for α above ~ 10 the model (satisfying the dark matter constraint) usually gives rise to a very light spectrum. So we choose to vary α from 4 to 10, which we think covers the most interesting region of KKLT-1. As can be seen in Eq.(1), $M_s\alpha$ controls the scale of the sparticle masses. To avoid those cases with very small masses which are excluded by the SUSY search limits as well as those with very large masses which are too heavy to be interesting at low luminosities, we take M_s to be in the range 25 GeV to 100 GeV. As usual $\tan \beta$ is taken to be from 1 to 50 (this is also used for all other models). As mentioned in Sec. IIB1, modular weights n_l , n_q and n_h are not fixed unless the string construction is

explicitly specified. So in order to be general we allow a continuous variation of the modular weights from 0 to 1/2 independently. In addition, for a practical step-by-step analysis of the scan of these modular weights, we divide the task of the complete scan into several pieces

$$\begin{cases}
\text{choice 1 : } 0 < n_h, n_q, n_l < 0.1 \\
\text{choice 2 : } 0 < n_h < 0.1, \quad 0 < n_l, n_q < 0.5 \\
\text{choice 3 : } 0 < n_h, n_q, n_l < 0.5
\end{cases}$$
(15)

The first choice corresponds to a perturbation of zero-modular-weight models with an uncertainty 0.1. The size of the error is somewhat arbitrary but will not affect any of our conclusions. The second choice is to allow for a large variation for quark and lepton modular weights. The third one should capture most of the cases with non-zero-modular weights, as those with modular weights beyond 0.5 are very likely to be excluded by the presence of tachyonic scalars. Clearly the first choice is a subset of the second one, which is in turn a subset of the third choice. Each choice is randomly sampled with 500 points.

For a given set of these parameters, we compute the corresponding soft terms at the unification scale⁸ and then evolve them using SOFTSUSY 2.0 [29] to the TeV scale to get the sparticle spectrum. The relic density of neutralino dark matter is calculated using MicrOMEGAs v1.3.6 [30]. Only models with $\Omega_{LSP}h^2 < 0.12$ are allowed by the relic density constraint. In the scan, models with tachyonic scalar masses as well as those which violate the chargino mass constraint ($m_{\tilde{\chi}_1} \geq 104$ GeV) or higgs mass constraint ($m_h \geq 114$ GeV) are rejected. We also impose a cutoff for stop mass less than 300 GeV. This is a convenient choice for simulation due to limited computing time and can be relaxed. It is clear this cutoff will remove the large cross section region of the footprint. However it will not affect the essential feature of the correlations between signatures and therefore, is not very important for our analysis.

For the KKLT-2 string-susy model, the scalar masses are generically heavy. To focus on this region, we take parameters ζ_i in the range from 0 to 1/6. ζ_i are assumed to be different for the lepton, quark and higgs fields but the same among different generations. The parameter α in this model can have a larger variation, which is taken from 4 to 20. The gravitino mass is chosen to vary from 1 TeV to 10 TeV.

For the footprint of LARGE Volume string-susy models, we generate sample points by varying M_3 from 400 GeV to 500 GeV and ϵ_i from 0 to 0.2. The lower value of M_3 is chosen so that the light higgs mass is above the current experimental limit. We have examined cases with M_3 less than 400 GeV and have found that almost all models generated are excluded by the higgs mass limit. The upper value set here is to avoid having too heavy a gluino.

 G_2 -MSSM models are generated by varying the parameters δ , P_{eff} and V_X in the following ranges:

$$-10 \le \delta \le 0$$
, $60 \le P_{eff} \le 84$, $V_{X,min} \le V_X \le V_{X,max}$, (16)

 $^{^{8}}$ we assume that these models give rise to gauge coupling unification as is suggested from the unification of couplings in the MSSM.

where $V_{X,min}$ and $V_{X,max}$ are functions of other parameters and can be found in [19]. We only consider the case in which Q - P = 3, since other cases either give rise to extremely heavy gravitinos or lead to AdS vacua [18]. The gauge couplings and Yukawa couplings at GUT scale are determined to match the low scale values. The RG evolution is carried out at 1-loop level with the "match and run" method to accommodate large scalar masses. Another consequence of heavy scalar masses is that the higgs bilinear parameter Z_{eff} is finely tuned to get EWSB breaking with correct Z boson mass. $\tan \beta$ is predicted in these vacua to be of $\mathcal{O}(1)$ [19].

IV. DISTINGUISHING STRING-SUSY MODELS FROM LHC SIGNATURES

Thus far we have constructed footprints for four (including two versions of KKLT) stringsusy models. The choices here are made basically to illustrate the results and techniques with limited computing. As data approaches and more string-susy models are added, more systematic calculations can be done. We have shown that footprints of string theories cover limited regions in LHC signature space, for understandable reasons. We now examine how to use these footprints to distinguish among string-susy models at the LHC. There have of course been discussions on how to distinguish different beyond-the Standard-Model constructions. There may be a signature which is sensitive to some features in the spectra and behaves differently for different models. For example, same-sign(SS) dileptons is a signature which has been widely discussed in the literature in distinguishing supersymmetric models from non-supersymmetric ones. However, this is often not very useful for distinguishing among various supersymmetric models, particularly those with an underlying high-scale. This is because the overall mass scale in each of these models is not fixed implying that the signatures can vary in a big range, which will wash out some simple correlations between (counting) signatures and features in spectrum. So it often happens that for most of the signatures the two scenarios overlap a lot and one can not tell them apart completely. But as we have seen there are correlations between certain pairs of signatures because of the structure of the underlying theory. This implies that it is likely that overlapping models of two different string-susy models do not overlap for other signatures. So systematically one would try to scan combinations of two signatures and check if the footprints are completely separated in the corresponding plots. This method was first explored in [1] and was found to be useful in some simple situations where the spectra of different string-susy models have big differences. For the string-susy models considered in this paper, the G_2 models can be distinguished from those of KKLT-1 and LARGE Volume by this method. For KKLT-1 and LARGE Volume models, we have found that no single 2D plot, i.e. no pair of signatures, can distinguish these models completely. However, as we will see below, these can still be distinguished by a combination of several 2D plots.

A. Extracting Correlations from 2D Plots

In this section we consider a more systematic way to extract correlations from signatures. Intuitively, one keeps track of the microscopic parameters associated with points in the overlap regions and eliminates parameters whenever they are not in an overlap region. In the remainder of this section and the next subsection we present some more technical and quantitative procedures. One should not lose sight of the essential point that one is distinguishing the theories by adding signature plots and keeping track of the microscopic parameters of the points in overlap region.

Conventionally one might try distinguishing classes of models by directly calculating the distance in the multi-dimensional signature space including a large number of signatures. A χ^2 -like quantity [3] could be defined with N_{sig} signatures as:

$$(\Delta S_{A_i B_j})^2 = \frac{1}{N_{sig}} \sum_{a=1}^{N_{sig}} \left(\frac{s_a^{A_i} - s_a^{B_j}}{\sigma_a^{A_i B_j}} \right)^2. \tag{17}$$

Here $s_a^{A_i}$ is signature a of model A_i and similarly for B. The quantity $\sigma_a^{A_iB_j}$ 9 characterizes the uncertainty in the a^{th} signature for the classes of models A and B. If the quantity $(\Delta S_{A_iB_j})^2$ is greater than the statistical fluctuation $(\Delta S_0)^2$ for all i's and j's, then one should be able to distinguish the two classes of models. However this method is not as effective as what we are proposing here. The average over a large number of signatures in Eq.(17) will diminish the difference between two classes of models if most of the signatures included are not effective in distinguishing them. A pre-selection of "useful" signatures could help but there is no systematic a priori way of knowing that. Indeed, this conventional method might not be useful at all, while the method we describe below always is.

Let us start with the following toy example. Suppose there are two signature plots a and b, each of which partially distinguishes footprint A and B. In other words, there is a sizable overlap between A and B for each plot, denoted as $(A \cap B)_a$ and $(A \cap B)_b$ respectively. If the signatures in plot a are correlated non-trivially with signatures in plot b, one would expect that at least some of the models of footprint A in the overlap $(A \cap B)_a$ can be differentiated from footprint B by signatures in the plot b, and so the set of models of footprint A in the overlap $(A \cap B)_a$ will have a smaller intersection with footprint B in $(A \cap B)_b$. In other words, $(A \cap B)_a \cap (A \cap B)_b$ is smaller than either $(A \cap B)_a$ or $(A \cap B)_b$. Therefore, the overlap region in 2D signature plots a or b does not imply a real degeneracy as it is lifted (at least partially) when more signatures are included. This idea is illustrated in Fig.8. In principle one can continue adding more signature plots and the overlap region is expected to be significantly reduced in the end. To technically realize the above idea, we need to make a few definitions to quantify the overlap of footprints in the following. First of all, we define the notion of degeneracy for two points $A_i \in A$ and $B_j \in B$:

Defn: Two points $A_i \in A$ and $B_j \in B$ are said to be degenerate in the 2D signature space (x, y) if the χ^2 -like quantity $(\Delta S_{A_iB_j})^2$ of the two points is smaller than the statistical

⁹ The definitions of $\sigma_a^{A_iB_j}$ and $(\Delta S_0)^2$ can be found below Eq.(18)

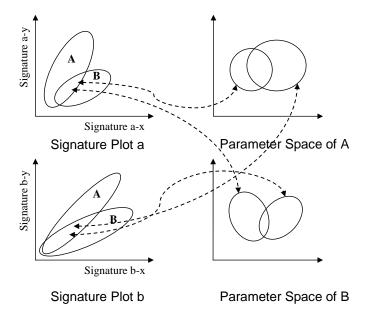


FIG. 8: Figure illustrating the idea that correlation between different signature plots can be used to reduce "false degeneracy".

fluctuation $(\Delta S_0)^2$.

For two signatures, $(\Delta S)^2$ in Eq.(17) becomes:

$$(\Delta S_{A_i B_j})^2 = \frac{1}{2} \left[\left(\frac{s_x^{A_i} - s_x^{B_j}}{\sigma_x^{A_i B_j}} \right)^2 + \left(\frac{s_y^{A_i} - s_y^{B_j}}{\sigma_y^{A_i B_j}} \right)^2 \right], \tag{18}$$

which characterizes the distance in the 2D signature space. Here $s_x^{A_i}$ is signature x of model A_i and similarly for others. The variance $(\sigma_x^{A_iB_j})$ is defined as $(\sigma_x^{A_iB_j})^2 = (\delta s_x^{A_i})^2 + (\delta s_x^{B_j})^2 + (f_i(s_x^{A_i} + s_x^{B_j})/2)^2$, where $f_i = 0.01$ for all counting signatures and $\delta s = \sqrt{s+1}$ for counting signatures [3]. ΔS_0 characterizes the statistical error, which is determined by simulating a large number of models with different random number seeds and taking the 95th percentile of the ΔS 's.

Defn: The model A_i is said to be degenerate with the entire footprint B with respect to the 2D signature plot (x, y) if there exists at least a model $B_j \in B$ such that A_i and B_j are degenerate.

In our convention the number of models of footprint A which are degenerate with B is denoted by $N_{A,B}$, similarly the number of models of footprint B which are degenerate with A is denoted by $N_{B,A}$. One can also notice that $N_{A,B}$ and $N_{B,A}$ are in general different, because the densities of models of the two footprints in the overlap region are different in general. So the overlap can be characterized by the algebraic mean $N(A,B) \equiv \frac{1}{2}(N_{A,B} + N_{B,A})$. It is clear from this definition that if the overlap calculated for footprints A and B vanishes, then the corresponding constructions can be completely distinguished.

As an example, we use the above method to distinguish (original) KKLT and LARGE Volume string-susy models. To estimate ΔS_0 , we have resimulated 100 KKLT models with

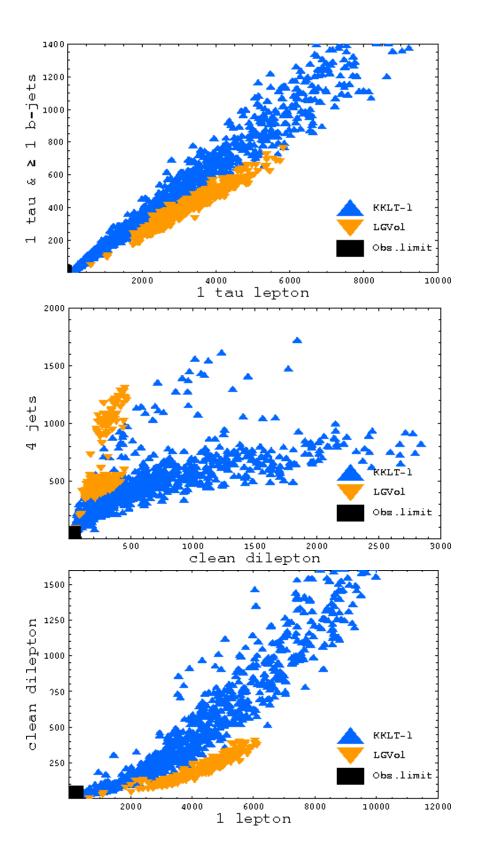


FIG. 9: Signature plots used for eliminating degeneracy between KKLT-1 (blue) and LGVol (orange) string-susy models. All models are simulated with $5fb^{-1}$ luminosity in PGS4 with L2 trigger. For the signature - "1 b-jet and 2 leptons" and "clean dilepton", there is no requirement of two hard jets. For all other signatures, there are at least two hard jets and large missing transverse energy. For each example, the points are generated by varying the microscopic parameters over their full

different random numbers and calculated ΔS for them. The 95th percentile of the ΔS distribution gives $\Delta S_0 \approx 1.5$. For different pairs of signatures ΔS_0 will vary by $\sim \pm 0.1$. As we have mentioned before, the first step of our strategy is to construct a large set of signature pairs. Without knowing which ones are better in distinguishing the two classes of models, one has to add all of them one by one. However to make the demonstration simple, we first do some trial-and-error analysis and find some signature plots which can partially distinguish these two classes of models. This will make the overlap decrease faster. For the present case, it is actually not difficult to find these if the P_T cut for jets is increased to 200 GeV based on the features in spectrum as explained in III C. Three of these plots are shown in Fig. 9. First we consider those models of KKLT-1 obtained with scan choice 1 (explained in (15)) and use the three plots in Figure 9 in our analysis:

- 1 tau lepton vs. 1 tau lepton and \geq 1 b-jets.
- clean dilepton¹⁰ vs. 4 jets.
- 1 lepton vs. clean dileptons.

Starting from the first plot, distances $\Delta S_{A_iB_j}$ between models in the two classes are calculated and those models of each class in the overlap region are selected. Then for the second plot the same procedure is performed except that those selected models in the previous plot are used instead. After that we will have a selection of models for each class which still remain in the overlap region for both plots. This procedure is carried out by adding more 2D signature plots until either the number of models in the intersection vanishes or does not decrease further. For the three plots in the order they are listed, we find the number of models in each class remained after each operation decreases monotonically as follows:

KKLT-1 (scan choice 1):
$$119 \rightarrow 4 \rightarrow 0$$

LARGE Volume: $237 \rightarrow 17 \rightarrow 0$. (19)

To test the stability of this sequence upon changes in ΔS_0 , we use $\Delta S_0 = 1.7$ and find a similar sequence

KKLT-1 (scan choice 1):
$$129 \rightarrow 5 \rightarrow 0$$

LARGE Volume: $259 \rightarrow 21 \rightarrow 0$. (20)

One can see that the number of models in the intersection quickly decreases as more plots are included. The final overlap of the two string-susy models is zero, which indicates that they can be distinguished readily at the LHC even with low luminosities. Furthermore, the models in the overlap region of each plot as well as the intersections of these overlap regions can be mapped back to the parameter space of the underlying string-susy model.

The exact number of models in the final overlap depends on how the parameter space is scanned and also how densely it is scanned. To make the method statistically robust, one

¹⁰ Clean dilepton signature is defined as the number of dilepton events with no hard jets passing the event selection cuts.

should sample the parameter space with a large enough number of points. Furthermore, to have a reliable count of models in the overlap region, the density of the points in the footprint should be large enough (at least for one class of models). In order to confirm that our result obtained with a sample of 500 points for these classes of models is robust¹¹, we include 1000 more points for the KKLT string-susy models (corresponding to scan choices 2 and 3)¹², and try to construct the sequence (19) again. We find the following:

KKLT-1 (All scan choices):
$$451 \rightarrow 37 \rightarrow 6$$

LARGE Volume: $477 \rightarrow 289 \rightarrow 69$. (21)

Now the number of models in the overlap does not vanish as before. However, when we consider three different combinations of the same signatures as that used earlier, namely:

- 1 lepton vs. 1 tau lepton
- 1 lepton vs. 4 jets
- 1 lepton vs. 1 tau lepton and \geq 1 b-jets

in addition to the previous combinations, the sequence again converge to zero as follows

KKLT-1 (All scan choices):
$$451 \rightarrow 37 \rightarrow 6 \rightarrow 4 \rightarrow 1 \rightarrow 0$$

LARGE Volume: $477 \rightarrow 289 \rightarrow 69 \rightarrow 11 \rightarrow 1 \rightarrow 0$. (22)

For $\Delta S_0 = 1.7$, we have

KKLT-1 (All scan choices):
$$506 \rightarrow 49 \rightarrow 10 \rightarrow 8 \rightarrow 7 \rightarrow 4$$

LARGE Volume: $488 \rightarrow 331 \rightarrow 114 \rightarrow 56 \rightarrow 18 \rightarrow 5$, (23)

which is almost as good as the sequence (22). We learn from the above that the overlap N(A, B) can increase with a denser scan of parameters. However if two classes of models can be distinguished intrinsically then N(A, B) will eventually vanish as more signature plots are included. If one finds N(A, B) approaches a nonzero value even when all possible combinations of signatures are included (for a given luminosity), then the two classes of models can not be distinguished completely. We will see in the next section that it is possible to define a quantity (which is independent of N(A, B)) to characterize the extent to which two classes of models can be distinguished.

In the above examples, the (close to) optimal set of useful signatures was arrived at by a judicious use of the trial-and-error method, i.e. by trying various signature plots sensibly based on the qualitative features of the classes of models described above. This procedure should work for other classes of models as well. However the main purpose of doing this here

¹¹ Of course, for different models, one would need to sample a different number of points in general depending on the structure of the microscopic parameter space.

¹² KKLT-1 models with these two choices have very similar footprints in the signature space and so including them will increase the footprint density significantly.

is to illustrate the idea without making it too complicated. In practice, a more systematic way to distinguish classes of models and pick out useful signatures is to simply add all kinds of possible signatures and keep track of the overlap. A sharp decrease in the overlap usually indicates that the signature pair just added is "good". In doing this, one does not need to know much about the features in the models and how they are related to the signatures, and so the procedure can be implemented in an automatic way. In future studies, the above procedure could be supplemented with modern statistical techniques such as neural networks, boosted decision trees, etc.

B. A Quantitative Definition of Distinguishibility

In this subsection, we propose a quantitative way to characterize the distinguishibility of two string-susy models. Let us denote the two classes of models as A and B. Suppose we can properly define a metric on signature space and hence the volume of the overlapping submanifold. Then a proper definition of the distinguishibility could be something like:

$$\eta(A,B) = 1 - \frac{S(A \cap B)}{2S(A)} - \frac{S(A \cap B)}{2S(B)},$$
(24)

where S(A) denotes the volume occupied by A in signature space and similarly for others. From this definition, we can see $0 \le \eta(A,B) \le 1$. Clearly if there is no overlap, i.e. $S(A \cap B) = 0$, $\eta(A,B)$ is equal to 1 indicating that the two footprints can be distinguished completely. On the other hand, if the two footprints completely overlap with each other, i.e. $S(A \cap B) = S(A) = S(B)$, then $\eta(A,B)$ is zero indicating that they can not be distinguished at all. For other intermediate cases, $\eta(A,B)$ is between 0 and 1 indicating partial distinguishibility, which is still useful depending on the location of experimental data. This will be discussed in detail in the next section.

In practice, the footprint is sampled by a large number of points instead of being a smooth manifold. For simplicity we will first assume that the points we sample are evenly distributed in the signature space, or more precisely, the number of point in a given region is proportional to its volume. This is not a realistic assumption, and we will relax it soon. In this case, the definition of η can be rewritten as:

$$\eta(A,B) = \lim_{N_A,N_B \to \infty} \left(1 - \frac{N_{A,B}}{2N_A} - \frac{N_{B,A}}{2N_B} \right),$$
(25)

where N_A and N_B are the total number of models in footprint A and B respectively, while $N_{A,B}$ and $N_{B,A}$ are as defined in the previous subsection. This gives a practical definition for distinguishibility. One can see that it is the *ratio* of the number of models in the overlapping region and the total number of models for a given class of models which contributes to its distinguishibility with another class of models. In practice, as long as N_A and N_B are large enough, the η value obtained will be very close to the formal value obtained after taking the limit to ∞ . For example, for KKLT-1 and LARGE Volume (with $\epsilon_0 = 0.2$), we found that none of the models in the two classes are degenerate. Thus, using this definition we get $\eta \to 1$, suggesting that the two string-susy models can be distinguished quite well.

In generic and more realistic cases the distribution of the models in the signature space is not flat. We now show that the definition (25) is a more natural one to use compared to definition (24) in such cases. It is reasonable to assume that sample points are generated uniformly in model parameter space. If we take into account the mapping to signature space, the points which sample the footprint are therefore not likely to be evenly distributed, but are rather assigned with a probability which is determined by the non-trivial mapping function. Suppose in the overlap region of two footprints this probability is small, then the number of points in that region is also comparatively small relative to the total number of points even though the volume of the overlap region is not. From definition (24), η and hence the distinguishibility would be small. However, using definition (25), η would and hence the distinguishibility would be large. This seems more natural since by assumption, it is much less likely to populate the overlap region compared to the non-overlap region by scanning model-parameters. Therefore we see that the definition (25) in terms of the number of points is quite reasonable and convenient in practice.

V. DISCUSSION AND CONCLUSION

In this paper, we have introduced the idea of constructing footprints of "string-susy models" (defined in Section IIA), and a general technique to distinguish different models by correlations of signatures. Focusing on four classes of string-susy models where calculations are reliable, our first major result is that they all give limited footprints in signature space. In addition, the LHC signatures of a particular class of models are sensitive to at least some of the underlying theoretical structure. This information is not only encoded in the values of signatures themselves but also in their correlations. Familiar inclusive dilepton and trilepton signatures are not very helpful in distinguishing among these string-susy models. However they can be distinguished by systematically adding and studying the pattern of signature plots and qualitatively understanding their origin. We have explicitly shown that the overlap area of two footprints becomes smaller and finally vanishes as more signatures plots are included.

Of course, it may be possible to recognize and interpret what is discovered rather easily. Our approach will worthwhile particularly even when superpartner masses and properties can not be untangled, and when degeneracies are present, so more traditional approaches work poorly. These methods can be applied with very limited data, and improved as integrated luminosities increase and more signatures become available.

The construction of footprints will of course be especially useful when the LHC data is available, which will appear as a box in the signature space. Suppose there are two string-susy models which can be distinguished from each other using the method described in this paper, with low luminosity data at the LHC. In the event of actual data, if the box corresponding to the data is far away from the footprints of both models, then both string-susy models are excluded. If the box is inside one of the footprints but not the other one, then the corresponding string-susy model is favored while the other is excluded. However,

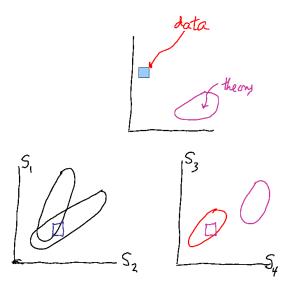


FIG. 10: Cartoon to illustrate the various possibilities once there is data. The square box denotes "data" while the elliptical regions denote slices of footprints of two string-susy models in two signature plots.

if the box lies in the overlap region of two footprints¹³, this means that both string-susy models are consistent with data at that particular luminosity. The cartoon in figure 10 illustrates the point. In this case, one can focus on all such models and both improve the theoretical constructions and add experimental observables and associated signature plots to distinguish among them further.

We have focussed on LHC counting signatures to demonstrate the methods. Other LHC data such as asymmetries (Figure 3), distributions in E_T , p_T , mass pairs, etc. all have limited footprints as well, and can be valuable. Dark matter detection and relic density involve the same physics and are meaningful to include for the signatures of an underlying microscopic theory (but not for a low-scale effective theory), as is $g_{\mu} - 2$. Adding these signatures will make the approach even more powerful. Since one has a full theory, it is meaningful to add them.

In the steps from high-scale string construction to LHC signatures, there are various kinds of uncertainties which were not yet considered properly. These include uncertainties in the ranges of the high-scale parameter space, uncertainties arising from the RG evolution codes, uncertainties arising from those in the SM parameters (the top quark mass for example), uncertainties in experimental constraints, as well as those arising from simulations event generation, parton showering and hadronization and detector simulation. One might worry whether these uncertainties will change the effectiveness of our method. Of course,

That is the box lies in the overlap of the entire footprints, not just some 2D footprints.

the boundary of the footprint will become fuzzy when these uncertainties are included. However since our analysis is based on the correlations of signatures, the distinguishibility of any two scenarios should not be affected. A better investigation of these issues is left for the future, as is the study of theories that give extended MSSMs for the visible sector. We expect the study of patterns of signature plots to be equally valuable in such cases.

Although particular classes of string-susy models with the same visible sector - the MSSM - can be distinguished, it may be more challenging to distinguish classes of string vacua with different matter and gauge spectra. If it turns out that there are some exotic fields beyond the MSSM which are light enough to be produced (on-shell or off-shell), there could be a substantial change in LHC signatures.

For all the string-susy models considered here, Electroweak symmetry breaking (EWSB) is accommodated but not predicted, in the sense that the μ parameter is chosen by hand so as to satisfy the EWSB condition. If one could find a natural dynamical mechanism giving EWSB in the future, it would be important to know what the effect of this natural mechanism would be on the pattern of signatures computed by just accommodating EWSB. The answer to this question is not definitive, since it depends on the particle spectrum and the exact mechanism. For example, within the context of the MSSM, it is well known that consistent EWSB requires a precise relation between the supersymmetric μ parameter and some soft supersymmetry breaking parameters. Therefore, one natural solution to the problem of EWSB is that this precise relation can be predicted or explained from the structure of a string-susy model. In this case, there would be no effect on the pattern of signatures. However, if the visible sector of the string-susy model consists of particles in addition to the MSSM which are light enough to be produced at the LHC and play a nontrivial role in the EWSB mechanism (additional U(1) models are an example), then the pattern of signatures could be affected significantly. Nevertheless, the methods described in the paper are still applicable.

We have proposed a new approach to relating collider data and an underlying theory. In the case where the underlying theory is a string construction with stabilized moduli and softly broken supersymmetry, we have seen that particular constructions give very limited footprints in signature space, and that LHC signatures for a particular class of vacua are sensitive to at least some of the underlying structure of the theory. Not all theories agree with data, and the subset that do can be distinguished by considering the pattern of a number of signatures. The software techniques needed to carry out such a program for a variety of constructions mostly already exist, and are improving. Our analysis needs to be extended in a number of directions, perhaps most by examining a number of constructions from different corners of string/M theory, with different compactifications, different ways of generating de Sitter vacua and different ways of breaking supersymmetry in a controlled manner.

We think that string theorists will learn about string theory by studying collider phenomenology. This has happened from studying the visible sector, particularly in heterotic and Type II toroidal constructions, and we expect it to happen from the study of superpartner properties. Given the large number of string vacua one could ask whether it is very unlikely that the ones we study could be like our vacuum? We think it is not so unlikely

because we do not study random string/M theory constructions - we select for study those that can give SM-like matter, softly broken $\mathcal{N}=1$ supersymmetry, dark matter, inflation, and so on. The approach described here may help us learn if we live in a string/M theory vacuum, and learn more about its properties.

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VI. APPENDIX: COUNTING SIGNATURES USED IN OUR STUDY

As we have discussed before, we are particularly interested in extracting information from the low luminosity data (5-10 fb^{-1}) corresponding to 1-2 year running of LHC. For this purpose, we will select a special set of counting signatures as our observables. A complete set of counting signatures can be found in [3]. Here we consider the following counting signatures:

- 1 lepton, OS dilepton, SS dilepton, trilepton, 1 tau lepton, 2 tau leptons, 3 tau leptons,
 OSSF dilepton, OSDF dilepton, SSSF dilepton, SSDF dilepton, OS dilepton(e,μ), SS dilepton(e,μ)
- 1 jet, 2 jets, 3 jets, 4 jets, 1 b-jet, 2 b-jets, 3 b-jets, 4 b-jets
- 2 leptons and 1 jet, 2 leptons and 2 jets, 2 leptons and 3 jets, 2 leptons and 4 jets
- 0 lepton and 2 b-jets, 0 lepton and \geq 3 b-jets, 1 lepton and \geq 2 b-jets, 2 leptons and 0 b-jet, 2 leptons and 1 b-jet, 2 leptons and 2 b-jets, 2 leptons and \geq 3 b-jets, 3 leptons and 1 b-jet
- 1 tau and \geq 1 b-jets, 1 tau and \geq 2 b-jets, 2 tau and \geq 2 b-jets, \geq 2 tau and 1 b-jet
- 1 positive lepton, 1 negative lepton, clean dilepton

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